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CHAOTIC HOMOGENEOUS POROUS MEDIA. 4. HEAT EXCHANGE IN A CELL

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An analytical study is made of a change in the intensity of heat exchange in a cell in passage from an ordered permeable system to a chaotic system.

One possible mathematical definition of a cell of a porous structure has been given in [1]. In the present work, we employ for the first time a structural method associated with the transformation of an ordered system where the notion of a cell is evident to a chaotic system. The object of investigation is a system of rods in longitudinal flow (Figs. 1 and 2). For such a model the characteristics of heat exchange in a cell in a steady-state stabilized regime are computed analytically. Furthermore, the analysis presented can be employed in calculating compact heat exchangers or nuclear-reactor cores.

Ordered System. In flow past parallel cylinders, there is no macrodispersion [2], and the laminarity condition makes it possible to eliminate transverse turbulent transfer; therefore, we will represent the cell in pure form without distortions. We consider two cases of heat exchange in the cell — with a Poiseuille velocity profile and a uniform (rod-shaped) profile — for an ordered system (Fig. 2a).

Poiseuille Flow. The hydrodynamic and heat-exchange problems for an ordered system of uniformly heated cylinders have been solved simultaneously in [3, 4]. The solution was obtained in the form of rapidly converging series. The Nu number was determined for $1.001 \le h/d_p \le 4.0$. It has been shown that when $h/d_p \ge 1.5$ any ordered configuration (triangular, square, hexagonal) of the system of cylinders can be replaced by an equivalent circular (ring) channel surrounding a cylinder. The error in calculating the Nu number does not exceed 5%. For the porosity Π the equality

$$\Pi = 1 - \frac{\pi}{2\sqrt{3}}\left(\frac{d_p}{h}\right)^2 , \tag{1}$$

holds, which yields the condition of existence of a *cell with a circular channel*: $\Pi \ge 0.6$. The dimension of the cell d_i is determined by the relation

$$\frac{d_p^2}{d_i^2} = 1 - \Pi . \tag{2}$$

Despite the geometric simplicity of the cell under study, the Nu number is calculated from a very cumbersome formula which is not given here. Below we will graphically represent the results of calculation of Nu (they have been obtained in [4]). The structure is homogeneous; therefore, the average value is $\langle \mathrm{Nu} \rangle$ = Nu and the average porosity is $\overline{\Pi} = \Pi$. The characteristic dimension is d_p; the hydraulic diameter for the system of rods in laminar flow is not used [4, 5].

Core Flow. Use is made of a cell with a circular channel of $\Pi \ge 0.6$ but with a uniform velocity profile (Fig. 3). The use of core flow for small Pe is substantiated in [6] in investigating heat exchange in microchannels.

The Nu number for the cell (see Fig. 3) can be determined in the regular manner: the simplest one-dimensional equations of transfer of heat are solved separately for the rod and the liquid, the conditions of conjugation at

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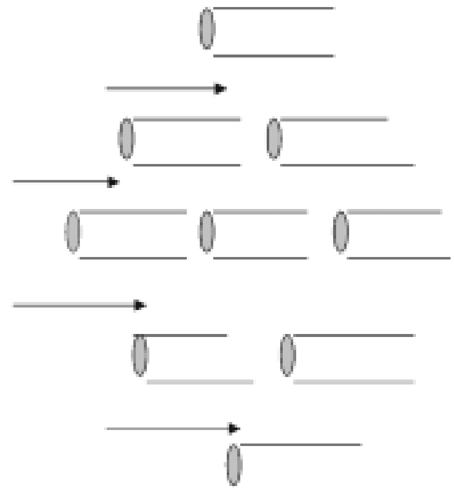

Fig. 1. Flow pattern.

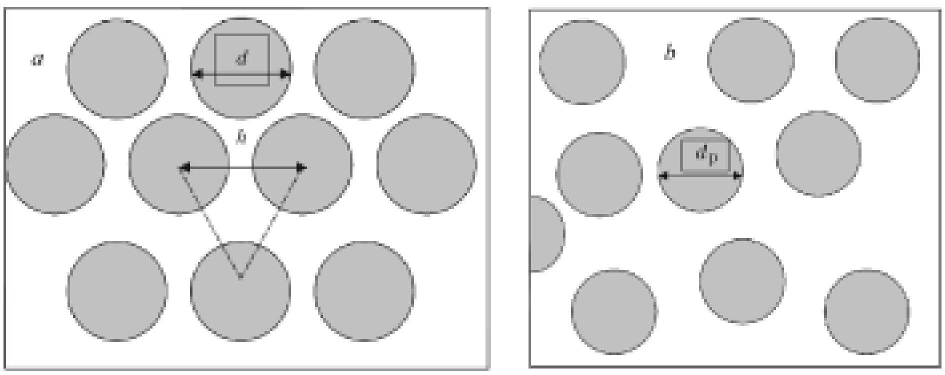

Fig. 2. Ordered (a) and chaotic (b) systems of cylinders.

the phase boundary and the heat insulation of the external boundary of the circular channel are employed, etc. But taking into account the stepwise character of heat-releasing and velocity fields, we can calculate Nu in a different manner. In this method, one uses a mathematical formalism of internal heat exchange in a porous medium. We introduce the characteristic function $\theta(r)$ equal to unity if r is in the liquid region and to zero if r is in the solid body. Then the equation of heat transfer for the entire cell $0 \le r \le r_i$ ($d_i = 2r_i$) has the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r \frac{\partial T}{\partial r}\right) + \frac{1-\theta}{1-\Pi} q_v + \frac{\theta}{\Pi} c\dot{m} \frac{\partial \bar{t}}{\partial x} = 0 \,, \quad \lambda = (1-\theta)\,\lambda_s + \theta\lambda_g \,. \tag{3}$$

The condition of a stabilized regime is $-c\dot{m}\dfrac{\partial \bar{t}}{\partial x} = q_v$. The boundary conditions are $\lambda \dfrac{\partial T}{\partial r}\Big|_{r=0, r=r_i} = 0$. The equation of convective heat exchange is

$$c\dot{m}\frac{\partial \bar{t}}{\partial x} = \alpha_v\,(\bar{t} - \bar{T}) \,. \tag{4}$$

The average temperatures $\bar{T}$ and $\bar{t}$ are determined from the formulas

$$\bar{T} = \frac{2}{r_i^2}\int_0^{r_i} \frac{1-\theta}{1-\Pi} Trdr \,, \quad \bar{t} = \frac{2}{r_i^2}\int_0^{r_i} \frac{\theta}{\Pi} Trdr \,. \tag{5}$$

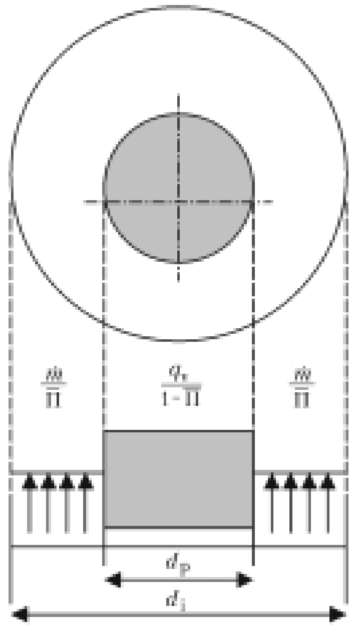

Fig. 3. Diagram of a thermal cell in a homogeneous structure of parallel heat-releasing cylinders in core flow of the coolant.

We integrate Eq. (3); then, with account for (4) and (5), we obtain

$$\frac{1}{\alpha_v} = \frac{2}{r_i^2}\int_0^{r_i}\frac{dr}{\lambda r}\left[\int_0^r\left(\frac{1-\theta}{1-\Pi}-\frac{\theta}{\Pi}\right)rdr\right]^2 . \tag{6}$$

Expression (6) is an analog of the Lyon integral for the coefficient of internal heat exchange.

Since the specific surface of the cylinders is $S_v = 4(1-\Pi)/d_p$ and $\alpha_v = S_v\alpha$ and $\lambda_s >> \lambda_g$, from (6) we have

$$\frac{1}{\mathrm{Nu}} = -\frac{\ln(1-\Pi)}{4\Pi^2} - \frac{1}{4\Pi} - \frac{1}{8} . \tag{7}$$

Chaotic System. Computation of the intensity of heat exchange in a chaotic system of rods in longitudinal flow, in addition to its being of independent importance, will enable us to represent a substantially modified cell. The inhomogeneity of the structure (Fig. 2b and Fig. 4) assumes the employment of probability methods for determination of the average values of the physical quantities, including the $\langle \mathrm{Nu} \rangle$ number.

Formulation of the Problem. In the system of disordered cylinders, the ordinary steady-state equations of heat exchange

$$\nabla\,(c\rho\mathbf{u}\tau + \mathbf{q}) = q_v\,, \quad \mathbf{q} = -\lambda\,\nabla\tau\,. \tag{8}$$

hold. The thermal conductivity λ of the structure is prescribed by the characteristic function $\theta(\mathbf{r})$ analogously to (3). The heat exchange is stabilized; therefore, we consider a plane problem and carry out all the averagings over the area; the fields of velocities and heat release are stepwise, such as those in core flow in the ordered system.

We average Eqs. (8) over the ring $S_r = 2\pi r\Delta r$ with the center at any point of the system (Fig. 4), using the method of [7]. The thickness of the ring is $\Delta r << r$, but for $r \geq R_0$, where R_0 is a fixed radius which is as large as is wished, the ring contains a set of cylinders. Therefore, from (8) we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_r r\frac{\partial t_r}{\partial r}\right) + \frac{\Pi_r}{\overline{\overline{\Pi}}}\,c\dot{m}\,\frac{\partial t_r}{\partial x} + \frac{1-\Pi_r}{1-\overline{\overline{\Pi}}}\,q_v = 0\,, \tag{9}$$

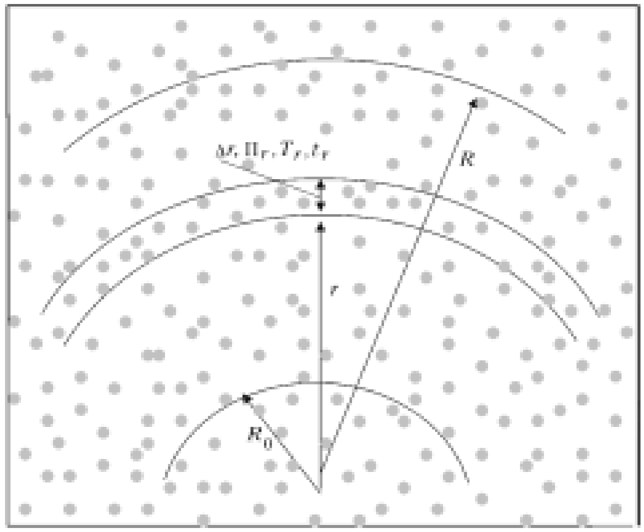

Fig. 4. On the derivation of the equation of heat exchange in a chaotic system of parallel cylinders.

$$\frac{1-\Pi_r}{1-\overline{\Pi}}\, q_{\mathrm{v}} = \alpha_{\mathrm{v},r}\,(T_r - t_r)\ . \tag{10}$$

The condition of a stabilized regime is

$$-\,c\dot{m}\,\frac{\partial t_r}{\partial x} = q_{\mathrm{v}}\ . \tag{11}$$

In Eqs. (9)–(11) and in Fig. 4, use is made of the following quantities:

$$\Pi_r = \frac{1}{S_r}\int\limits_r^{r+\Delta r}\int\limits_0^{2\pi} \theta\,(r,\,\varphi)\; r dr d\varphi\ , \tag{12}$$

$$t_r = \frac{1}{\Pi_r S_r}\int\limits_r^{r+\Delta r}\int\limits_0^{2\pi} \theta\,(r,\,\varphi)\;\tau\,(r,\,\varphi)\; r dr d\varphi\ , \tag{13}$$

$$T_r = \frac{1}{(1-\Pi_r)\,S_r}\int\limits_r^{r+\Delta r}\int\limits_0^{2\pi} [1-\theta\,(r,\,\varphi)]\;\tau\,(r,\,\varphi)\; r dr d\varphi\ . \tag{14}$$

The parameters λ_r and $\alpha_{\mathrm{v},r}$ will be determined below. The variables Π_r, t_r, and T_r enable us to compute the true average values $\overline{\Pi}(R)$, $\overline{t}(R)$, and $\overline{T}(R)$. Indeed, for $\Delta r << R_0$ it is easy to show that

$$\overline{\Pi}\,(R) = \frac{1}{\pi\,(R^2 - R_0^2)}\int\limits_{R_0}^{R}\int\limits_0^{2\pi} \theta\,(r,\,\varphi)\; r dr d\varphi = \frac{2}{R^2 - R_0^2}\int\limits_{R_0}^{R} \Pi_r\, r dr\ , \tag{15}$$

$$\overline{t}\,(R) = \frac{1}{\pi\overline{\Pi}\,(R)\,(R^2 - R_0^2)}\int\limits_{R_0}^{R}\int\limits_0^{2\pi} \theta\,(r,\,\varphi)\;\tau\,(r,\,\varphi)\; r dr d\varphi = \frac{2}{\overline{\Pi}\,(R)\,(R^2 - R_0^2)}\int\limits_{R_0}^{R} \Pi_r t_r r dr\ , \tag{16}$$

$$\bar{T}(R) = \frac{1}{\pi\,[1 - \bar{\Pi}(R)]\,(R^2 - R_0^2)} \int_{R_0}^{R} \int_{0}^{2\pi} [1 - \theta(r, \varphi)]\, \tau(r, \varphi)\, r dr d\varphi =$$

$$= \frac{2}{\pi\,[1 - \bar{\Pi}(R)]\,(R^2 - R_0^2)} \int_{R_0}^{R} (1 - \Pi_r)\, T_r r dr\,. \tag{17}$$

We have selected the ring S_r as the object of averaging since, first, in this case we can easily find the average values of the true temperatures of the cylinders and the coolant and, second, in such an averaging, the heat sources and sinks on the plane are not deformed and retain their position.

Further algorithm of solution of the problem is as follows: we solve Eqs. (9) and (10) determining the temperatures t_r and T_r, find the average $\bar{\Pi}(R)$, $\bar{t}(R)$, and $\bar{T}(R)$ from formulas (15)–(17), and compute the difference $\bar{T}(R) - \bar{t}(R)$ which is averaged over the ensemble for the entire plane $R \to \infty$.

Solution of the Problem. From Eq. (9) with account for (11) we obtain

$$\frac{1}{r} \frac{\partial}{\partial r}\left(\lambda_r r \frac{\partial t_r}{\partial r}\right) = \left(\frac{\Pi_r}{\bar{\bar{\Pi}}} - \frac{1 - \Pi_r}{1 - \bar{\bar{\Pi}}}\right) q_{\text{v}}\,. \tag{18}$$

Integrating (18), we have

$$t_r = q_{\text{v}} \int_{R_0}^{r} \left[\int_{R_0}^{r} \left(\frac{\Pi_r}{\bar{\bar{\Pi}}} - \frac{1 - \Pi_r}{1 - \bar{\bar{\Pi}}} \right) r dr \right] \frac{dr}{\lambda_r r} + C_1 \int_{R_0}^{r} \frac{dr}{\lambda_r r} + C_2\,. \tag{19}$$

The temperature of the cylinders in the ring T_r is determined from (10). From formulas (15)–(17) we find the difference $\bar{T}(R) - \bar{t}(R)$, using, where necessary, the integral mean-value theorem:

$$\bar{T}(R) - \bar{t}(R) = \frac{q_{\text{v}}\,[1 - \Pi_r(r_0)]}{(1 - \bar{\bar{\Pi}})\,\alpha_{\text{v},r}(r_0)} - \frac{2 q_{\text{v}}}{[1 - \bar{\Pi}(R)]\,(R^2 - R_0^2)} \int_{R_0}^{R} \left[\int_{R_0}^{r} \left(\frac{\Pi_r}{\bar{\bar{\Pi}}} - \frac{1 - \Pi_r}{1 - \bar{\bar{\Pi}}} \right) r dr \int_{R_0}^{r} (1 - \Pi_r)\, r dr \right] \frac{dr}{\lambda_r r} -$$

$$- \frac{2 C_1}{[1 - \bar{\Pi}(R)]\,(R^2 - R_0^2)} \int_{R_0}^{R} \left[\int_{R_0}^{r} (1 - \Pi_r)\, r dr \right] \frac{dr}{\lambda_r r} + \frac{2 q_{\text{v}}}{\bar{\Pi}(R)\,(R^2 - R_0^2)} \int_{R_0}^{R} \left[\int_{R_0}^{r} \left(\frac{\Pi_r}{\bar{\bar{\Pi}}} - \frac{1 - \Pi_r}{1 - \bar{\bar{\Pi}}} \right) r dr \int_{R_0}^{r} \Pi_r r dr \right] \frac{dr}{\lambda_r r} +$$

$$+ \frac{2 C_1}{\bar{\Pi}(R)\,(R^2 - R_0^2)} \int_{R_0}^{R} \left[\int_{R_0}^{r} \Pi_r r dr \right] \frac{dr}{\lambda_r r}\,, \quad R_0 \le r_0 \le R\,. \tag{20}$$

In porosity averaging, the terms with C_1 cancel out; therefore, they will not be taken into account in what follows. For the convenience of computations we introduce the following notation:

$$\bar{\Pi}(r) = \frac{2}{r^2 - R_0^2} \int_{R_0}^{r} \Pi_r r dr\,,$$

then for the two remaining integrals in (20) we obtain

$$\frac{2 q_{\text{v}}}{\bar{\Pi}(R)\,(R^2 - R_0^2)} \int_{R_0}^{R} \left[\int_{R_0}^{r} \left(\frac{\Pi_r}{\bar{\bar{\Pi}}} - \frac{1 - \Pi_r}{1 - \bar{\bar{\Pi}}} \right) r dr \int_{R_0}^{r} \Pi_r r dr \right] \frac{dr}{\lambda_r r} -$$

$$-\frac{2q_v}{[1-\overline{\Pi}(R)](R^2-R_0^2)}\int_{R_0}^{R}\left[\int_{R_0}^{r}\left(\frac{\Pi_r}{\overline{\Pi}}-\frac{1-\Pi_r}{1-\overline{\Pi}}\right)rdr\int_{R_0}^{r}(1-\Pi_r)\,rdr\right]\frac{dr}{\lambda_r r}=$$

$$=\frac{q_v}{2\overline{\Pi}(1-\overline{\Pi})(R^2-R_0^2)}\int_{R_0}^{R}\left\{\frac{[\overline{\Pi}(r)]^2-\overline{\Pi}\,\overline{\Pi}(r)-\overline{\Pi}(R)\,\overline{\Pi}(r)+\overline{\Pi}\,\overline{\Pi}(R)}{[1-\overline{\Pi}(R)]\,\overline{\Pi}(R)}\right\}\frac{(r^2-R_0^2)^2\,dr}{\lambda_r r}\,. \quad (21)$$

Averaging (20) with account for (21), we have

$$\langle\overline{T}(R)-\overline{t}(R)\rangle=\langle\overline{T}(R)\rangle-\langle\overline{t}(R)\rangle=\frac{q_v}{\langle\alpha_{v,r}\rangle}+\frac{q_v}{2\overline{\Pi}^2(1-\overline{\Pi})^2(R^2-R_0^2)\langle\lambda_r\rangle}\int_{R_0}^{R}(r^2-R_0^2)^2\,D[\overline{\Pi}(r)]\frac{dr}{r}\,. \quad (22)$$

According to [8], the dispersion of the porosity is $D[\overline{\Pi}(r)] = \overline{\Pi}(1-\overline{\Pi})\frac{d_D^2}{4(r^2-R_0^2)}$.

Integrating (22) and passing to the limit when $R \to \infty$, we obtain

$$\langle\overline{T}\rangle-\langle\overline{t}\rangle=\frac{q_v d_D^2}{16\overline{\Pi}(1-\overline{\Pi})\langle\lambda_r\rangle}+\frac{q_v}{\langle\alpha_{v,r}\rangle}\,. \quad (23)$$

For the average coefficient of internal heat exchange $\langle\alpha_v\rangle$ the relation

$$q_v=\langle\alpha_v\rangle(\langle\overline{T}\rangle-\langle\overline{t}\rangle)\,,$$

which is analogous to (10), holds; therefore, we finally obtain

$$\frac{1}{\langle\alpha_v\rangle}=\frac{d_D^2}{16\overline{\Pi}(1-\overline{\Pi})\langle\lambda_r\rangle}+\frac{1}{\langle\alpha_{v,r}\rangle}\,. \quad (24)$$

The dispersion diameter d_D formally introduced in [8] as the linear dimension characterizing the geometric dispersion of the porosity $D[\Pi]$ acquires a physical meaning — the heat exchange in an inhomogeneous permeable structure substantially depends on this parameter.

Analysis of the Results for the Chaotic System. A thermal cell in a chaotic system of similar cells occupies the entire plane in contrast to the ordered system where the isolated cell surrounds the cylinder.

The most reliable and widespread formula for the transverse effective thermal conductivity $\langle\lambda_r\rangle$ is that of Maxwell and Wiener [9, 10]. The thermal conductivity of heated cylindrical inclusions is $\lambda_s >> \lambda_g$; in this case the Maxwell–Wiener formula has the form

$$\langle\lambda_r\rangle=\frac{2-\overline{\Pi}}{\overline{\Pi}}\lambda_g\,. \quad (25)$$

We have

$$\langle\alpha_v\rangle=\langle S_v\rangle\langle\alpha\rangle\,,\quad \mathrm{Nu}_v=\frac{\alpha_v d_p^2}{\lambda_g} \quad (26)$$

The average specific surface is

$$\langle S_v\rangle=4(1-\overline{\Pi})/d_\eth\,,\quad \langle\mathrm{Nu}_v\rangle=4(1-\overline{\Pi})\langle\mathrm{Nu}\rangle\,. \quad (27)$$

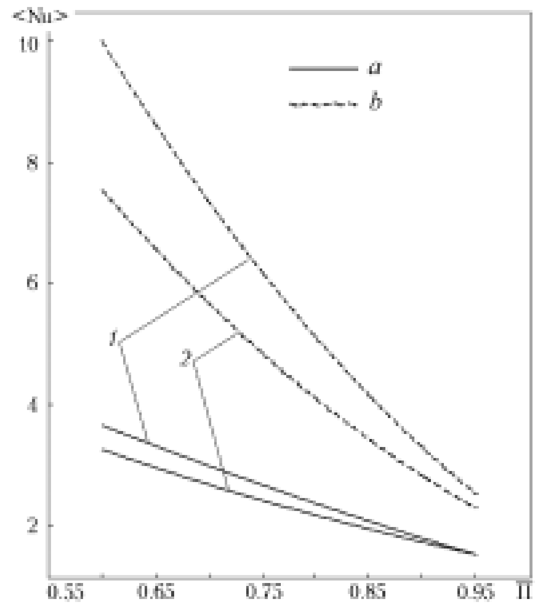

Fig. 5. Intensity of heat exchange in chaotic (a) and homogeneous (b) systems of cylinders: 1) core flow of the coolant; 2) Poiseuille flow.

From formula (24), we finally obtain

$$\frac{1}{\langle \mathrm{Nu} \rangle} = \frac{1}{4\,(2-\overline{\Pi})} \left(\frac{d_D}{d_{\eth}} \right)^2 + \frac{1}{\langle \mathrm{Nu}_r \rangle} . \tag{28}$$

Here $\langle \mathrm{Nu}_r \rangle$ is the average local number Nu in the ring $S_r = 2\pi r \Delta r$. In [2], it has been shown that we have $d_p = d_D$ for $\Pi \geq 0.6$; therefore, we obtain

$$\frac{1}{\langle \mathrm{Nu} \rangle} = \frac{1}{4\,(2-\Pi)} + \frac{1}{\langle \mathrm{Nu}_r \rangle} . \tag{29}$$

In deriving Eqs. (9)–(29), we have disregarded the molecular thermal conductivity of the coolant along the flow as compared to the convective component.

Average Local Coefficients of Heat Exchange $\langle \alpha_r \rangle$ *and* $\langle \alpha_{v,r} \rangle$ *and* $\langle \mathrm{Nu}_r \rangle$ *and* $\langle \mathrm{Nu}_{v,r} \rangle$ *Numbers for the Chaotic System.* It is axiomatic that the heat-transfer coefficient of an inhomogeneous structure $\langle \alpha_r \rangle$ differs from the quantity α in an ordered system. We note that the ring $S_r = 2\pi r \Delta r$ contains a fairly large number of cylinders, which is necessary for averaging of the heat-exchange parameters.

One model determining the values of $\langle \alpha_r \rangle$, $\langle \alpha_{v,r} \rangle$, $\langle \mathrm{Nu}_r \rangle$, and $\langle \mathrm{Nu}_{v,r} \rangle$ and developed by V. I. Subbotin and V. V. Kharitonov [11, 12] is based on the relation of the hydraulic resistance to the coefficient of internal heat exchange of a porous structure. But this semiempirical theory has been constructed for a developed turbulent flow and will not be used for the case under study.

Another, more general, model explaining the difference of $\langle \alpha_r \rangle$ for a chaotic system from the heat-transfer coefficient α in an ordered structure is the theory of convective diffusion or small-scale dispersion, whose mechanics and mathematical apparatus have been investigated in the works of V. N. Nikolaevskii, Yu. A. Buevich, M. I. Shvidler, and M. Kaviany (see [13]). A bibliography of earlier works on dispersion theory can be found in [14, 15]. The assumption that the average of the products of velocity and concentration (temperature) pulsations is an additional pulsation material (heat) flux proportional to the gradient of the average concentration (temperature) is employed in the theory of convective diffusion in averaging local equations of transfer in an individual pore (small scales) but with allowance for fluctuations of the velocity and concentration fields. The proportionality factor is the convective-diffusion tensor dependent on the vector of the average velocity.

Small-scale dispersion effects act in macroregions where there are correlations between the hydrodynamic parameters, i.e., the linear dimensions of such portions are smaller than the macromixing length $\Delta r < l_M$ [2]. Employing the experimentally determined relations between the hydraulic resistance, the average velocity of the flow, and the heat-transfer coefficients, Buevich and Ustinov [16] have shown that the difference of $\langle\alpha_r\rangle$, $\langle\alpha_{v,r}\rangle$, and $\langle Nu_r\rangle$ from the corresponding quantities for a homogeneous structure does not exceed 30%. Since the decrease in the total average intensity of heat exchange in a chaotic structure of parallel cylinders is considerable (a decrease of 280% for the core flow and of 230% for the Poiseuille flow when $\overline{\Pi}$ = 0.6, Fig. 5, formula (29)), for a not very rough evaluation we can set $\langle\alpha_r\rangle = \alpha$, $\langle\alpha_{v,r}\rangle = \alpha_v$, and $\langle Nu_r\rangle$ = Nu. We note that for small Pe numbers the increase in the effective thermal conductivity will be insignificant and tending to zero, which enables us to employ the Maxwell–Wiener formula (25).

Thus, the chaotic nature of the structure of cylinders in longitudinal laminar flow decreases the intensity of stabilized heat exchange 2 to 3 times.

It is clear that, in contrast to the thermal cell, the *hydrodynamic cell changes only slightly* — within the small-scale-dispersion model — in passage to an inhomogeneous structure.

In the *chaotic spherical packing*, we have $\overline{\Pi}$ = 0.37–0.4 and the hydraulic diameter is equal to the dispersion diameter [8]. Employing the Maxwell formula for spherical inclusions [9] and dependence (24), we can easily show that the intensity of heat exchange for the laminar flow changes only slightly and is totally determined by correlation corrections for the small-scale inhomogeneity.

In all the examples considered, we *have disregarded macrodispersion*; at a later time, we will show a substantial influence of the continuum of the macrodispersion M [2] on the formation of temperature fields and the intensity of internal heat exchange.

NOTATION

h, distance between the centers of the cylinders in an ordered system, m; r and d, radius and diameter, m; S, area, m^2; Π, dimensionless porosity of an ordered structure, coincident with the average porosity $\overline{\Pi}$ of a disordered structure; Nu and $\langle$Nu$\rangle$, dimensionless Nusselt numbers for a homogeneous system and a chaotic system; Pe, dimensionless Péclet number; x, r, φ, components of the cylindrical coordinates, m, m, rad; $\theta(r)$, $\theta(\mathbf{r})$, and $\theta(r, \varphi)$, dimensionless characteristic functions; λ, thermal conductivity, W/(m·K); T and t, temperatures of the cylinders and the coolant, K; q_v, power of the specific heat sources, W/m^3; c, specific heat at constant pressure of the coolant, J/(kg·K); $\dot{m}$, specific flux of the coolant, kg/(m^2·^{o}C); α, heat-transfer coefficient, W/(m^2·K); α_v, coefficient of internal heat exchange, W/(m^3·K); ρ, density of the coolant, kg/m^3; $\mathbf{u}$, velocity field of the coolant, m/sec; $\mathbf{q}$, heat flux, W/m^2; τ, continuous temperature field of the entire structure, K; Δr, thickness of the ring, m; R_0 and R, initial and final radii of the ring of averaging of the entire system, m; C_1 and C_2, integration constants; $\overline{\Pi}(R)$, $\overline{t}(R)$, and $\overline{T}(R)$, average values of the corresponding quantities for the ring $R_0 \le r \le R$; r_0, intermediate radius from the integral theorem, m; $D[\overline{\Pi}(r)]$, dimensionless dispersion of the porosity; λ_r, effective transverse thermal conductivity of the chaotic system of parallel cylinders with a coolant, W/(m·K); l_M, macromixing length, m; M, macrodispersion continuum; $\langle\overline{T}\rangle$, value of the quantity T, for example, of the temperature averaged over the area (bar) and the ensemble of configurations of systems of chaotic parallel cylinders (angle brackets), for the porosity $\overline{\Pi} = \langle\Pi\rangle$. Subscripts: p, particle; r, relating to the quantities averaged in the ring $S_r = 2\pi r\Delta r$; i, relating to the quantities in the thermal cell; s, for solid cylinders; g, for the coolant; v, relating to the quantities in unit volume; D, relating to the quantities determined by the dispersion of the porosity $D[\Pi]$; M, relating to the values of the macrodispersion continuum.

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